

The Actuarial Utility of Weather and Climate Predictions

PAUL ROEBBER¹

University of Wisconsin-Milwaukee

VYTARAS BRAZAUSKAS²

University of Wisconsin-Milwaukee

SERGEY KRAVTSOV³

University of Wisconsin-Milwaukee

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Abstract. The weather and climate systems are necessarily complex, being non-linear, non-stationary, and composed of multiple, interacting spatial and temporal scales. Additionally, the observational networks do not sample all conditions at all the necessary space and time scales to resolve these features. A primary focus in weather and climate prediction has been to first build simple, and then increasingly complex numerical models to solve the basic fluid dynamical equations and the many physical process models that are required to represent these motions. A second, important focus has been to augment or extend these numerical models with statistical models or statistical-dynamical hybrid models to improve predictive performance at the time and space scales of interest to end users. A third focus is to use modeling systems to produce dynamically consistent reanalyses of observational data, effectively extending the available observations. This paper provides a review of the basics of these efforts, and ends with a focus on possible implications of these methods for actuarial work. In particular, it describes the potential utility of weather and climate predictions in enhancing inputs of the Actuaries Climate Index and in estimating expected payoffs of weather derivatives.

¹ Paul Roebber, Ph.D., Distinguished Professor, Atmospheric Science Group, Department of Mathematical Sciences, University of Wisconsin-Milwaukee, P.O. Box 413, Milwaukee, WI 53201, USA. *e-mail:* roebber@uwm.edu

² CORRESPONDING AUTHOR: Vytautas Brazauskas, Ph.D., ASA, Professor and Co-Director, Actuarial Science Program, Department of Mathematical Sciences, University of Wisconsin-Milwaukee, P.O. Box 413, Milwaukee, WI 53201, USA. *e-mail:* vytaras@uwm.edu

³ Sergey Kravtsov, Ph.D., Professor, Atmospheric Science Group, Department of Mathematical Sciences, University of Wisconsin-Milwaukee, P.O. Box 413, Milwaukee, WI 53201, USA. *e-mail:* kravtsov@uwm.edu

1 Introduction

Weather and climate are distinct concepts that are often confused. In atmospheric sciences, there is an expression that nicely defines the difference between them: “climate is what you expect, weather is what you get.” In other words, climate is the statistical aggregate of individual weather events over some appropriate time interval. This also makes clear that higher order distribution moments (such as the standard deviation) are integral to a complete characterization of climate.

The motions of the atmosphere are non-linear, and composed of multiple, interacting spatial and temporal scales: a tropical storm, which is itself driven by interactions between the sea-surface (temperature), thunderstorms, and winds at upper levels of the atmosphere, can begin to recurve towards the northeast and force an adjustment of the upper level jet stream, changing the temperatures over North America and Europe for days-to-weeks (Archambault *et al.*, 2013; 2015). Natural cycles in the atmosphere ebb and flow over weeks-to-months-to-years-to decades, adding an additional layer of complexity to the climate system.

The non-linearity of atmospheric flows leads to a phenomenon known as sensitive dependence on initial conditions, meaning that even for a perfect model of its behavior, as long as one does not know to perfect precision the current state of all atmospheric variables, a forecast will depart from the truth at some future time (Lorenz, 1963). The rate at which these non-linearities overcome predictions varies depending on the phenomena of interest, from minutes-to-hours for thunderstorms to days-to-weeks for air temperature. In reality, while weather and climate models have improved, they still contain error, and the observations themselves contain error and are sampled at less than infinite spatial resolution. Thus, the one prediction that can be made with 100% certainty is that there will always be forecast uncertainty.

Given that climate represents an aggregation of high frequency (in both space and time) properties of the atmosphere, one might suppose that climate predictability should be confined to the same lead times as the weather. This is not so, because there is predictability of a second type, due to the predictable response of a given climate subsystem, say the atmosphere, to external forcing (e.g. greenhouse gases, or sea surface temperature signals stemming from long-term intrinsic ocean dynamics

etc.). In order to gain some fundamental understanding of these complex behaviors, considerable effort over decades has been expended by researchers to first build simple, and then increasingly complex numerical models to solve the basic fluid dynamical equations and the many physical process models that are required to represent these motions. Owing to model error, a second, important focus has been to augment or extend these numerical models with statistical models or statistical-dynamical hybrid models to improve predictive performance. Some of these models can also be used to produce dynamically consistent reanalyses of observational data, effectively using understanding of meteorological dynamics to extend the spatial density of available observations (which are taken at irregular spatial intervals).

In order to clarify these points, Section 2 will outline details of the climate system, including the nature of the time and space scales involved and how these depend on the nature of the physics, how weather and climate predictions are currently accomplished and their accompanying limitations, and how the observations that make up the understanding of the current atmospheric state are used to facilitate these predictions. Section 3 describes how modern meteorological understanding can be leveraged effectively in actuarial work. Section 4 provides a summary.

2 The Weather and Climate System

2.1 Time and Space Scales

Meteorological flows can roughly be classified using a log-log relationship between spatial scale on the one hand, and duration on the other (Fujita, 1981). Starting from smallest to largest scale phenomena of predictive interest, there are:

- tornadoes (100s of meters, minutes),
- wind micro- and macrobursts (kilometers, 10s of minutes),
- supercell thunderstorms (10s of kilometers, hours),
- tropical and extratropical cyclones (hundreds to thousands of kilometers, days to weeks),
- jet stream (thousands of kilometers, weeks),

- planetary scale or “long” waves (10s of thousands of kilometers, several weeks).

This list can be extended to include the processes operating on even longer time scales associated with internal ocean or coupled ocean–ice–atmosphere dynamics, but the discussion of these phenomena is beyond the scope of this presentation. Note that the scaling relationship above most emphatically does not apply to the strength or weather impact of the systems themselves, which can levy destructive force across this entire range.

Based on these physical relationships, and partly as an artifact of history, meteorologists tend to organize their thinking about these systems starting from the largest scales and working downward. In this, there is the implicit understanding that the largest scales pass energy downscale but also that considerable upscale energy transfer occurs. An example of the latter is the case in which a collection of thunderstorms develop along the warm front of an extratropical cyclone, releasing heat at mid-levels of the atmosphere as moisture condenses, thus warming the environment and changing the circulations and dynamics of the larger-scale system in the process. Since the predictability of an individual thunderstorm is on the order of minutes, this upscale energy transfer can result in fast degradation of forecast skill at the larger scales. Further, this scale description is approximate: the energy spectra of atmospheric flows shows no true scale separation.

Nonetheless, the expectation is that a good forecast of the larger scale is a necessary condition for making forecasts at smaller scales, and further, that for a given large scale pattern, the range of possible outcomes becomes restricted. Despite this, however, attempts at analog forecasting, which seek to find flows similar to today’s in the historical record, have not proven to be useful, primarily owing to the large variability contained in flows that might be considered grossly similar. In the following section, we will detail the range of models that have been developed in order to address atmospheric prediction.

2.2 Weather and Climate Predictions

In the earliest days of weather forecasting, a rather small set of approximately simultaneous observations were collected and plotted. Using relatively simple conceptual models of meteorological systems, projections for a short time into the future were made. As scientific understanding grew, so did the

ability to project these future states using mathematical models built by solving approximations of the fluid dynamical equations on a grid. The primary limitation in that effort, aside from the availability of observations to initialize the models, was the lack of sufficient computational power.

As computer power has grown, the models have become increasingly complex and sophisticated. Today, these models solve the so-called primitive equations, which involve a continuity equation (representing the conservation of mass), a form of the Navier-Stokes equation to represent conservation of momentum, and a thermodynamic equation describing changes in temperature in relation to heat sources and sinks. These equations allow for predictions of the horizontal and vertical components of the wind and temperature over a spatial domain. As anyone familiar with weather knows, this is an incomplete description: an understanding of phase changes in water substance is essential for predictions of sensible weather (clouds and amount and type of precipitation). This requires moisture conservation equations and equations describing microphysical properties in clouds. Furthermore, heat sources and sinks include significant contributions from radiative processes, which also must be modeled, and which in turn require considerable information about processes in the lowest layers of the atmosphere and heat, moisture and momentum exchanges between that layer and the ground surface. Mountainous terrain can add additional scale-dependent complexity since the resolution of the model will determine how well the effects of such features can be represented. Additionally, so-called “sub-grid scale” processes, that is, those that cannot be directly represented because of the resolution of the model must have their effects on the resolved scales be parameterized. For example, many weather prediction models and all climate models are too coarse to represent cumulus clouds.

The atmosphere covers the globe. Therefore, the models must either cover the entire globe with its grid or else default to representing some region of the globe, with the missing information being passed through the horizontal boundaries. Many weather prediction models are regional, with the missing information being passed to them from another, global model, which typically will solve the equations with lower spatial resolution. Some models do not solve the equations on a grid, but rather use spectral methods, where the solutions are represented as a sum of “basis functions.” Spectral rather than grid-based methods are often used in global models, where their computational efficiency is particularly necessary given finite computing resources. In recent years, there has been a convergence in approaches

such that climate models are quite similar in their construction to global weather prediction models.

In meteorology, the models described above, which are largely based on first principles except for (often *ad hoc*) parameterizations of unresolved, subgrid-scale processes, are referred to as dynamical models. In order to gain further improvements in dynamical models' forecast skill, many post-processing statistical techniques have been used to treat their predictions by utilizing available observations or forecasts of multiple dynamical models in an attempt to correct model biases. These techniques included multiple linear regression (Glahn and Lowry, 1972), artificial neural networks (e.g., Koizumi, 1999; Kuligowski and Barros, 2001), evolutionary programming (Bakhshai and Stull, 2009; Roebber, 2010), quantile mapping (Scheuerer and Hamill, 2015), ensemble Kalman filtering (e.g., Houtekamer and Mitchell, 1998), Bayesian Model Averaging (Raftery *et al.*, 2005), and Bayesian Model Combination (Roebber, 2015), either alone or along with bias correction (e.g., Cui *et al.*, 2012), among others. Furthermore, purely statistical, empirical stochastic models of weather and climate, which are entirely data driven and do not utilize physical principles at all, have also been developed and showed success in predicting some of the important weather and climate phenomena with the forecast skill often comparable with that of dynamical models (e.g., Penland, 1989; Penland and Ghil, 1993; Winkler *et al.*, 2001; Kravtsov *et al.*, 2005; Kondrashov *et al.*, 2005, and many others). Empirical models necessarily concentrate on a limited subset of climatic fields, due to a typically short length or limited spatial coverage (or resolution) of available observational records, and are often relatively low-dimensional. However, recent work showed that high-resolution empirical modeling across a wide range of weather and climate time scales can be feasible (Kravtsov *et al.*, 2016), arguably making these models a competitive tool for weather and climate forecasting, with the numerical cost being but a small fraction of that associated with state-of-the-art dynamical models.

2.3 Analyses

Meteorological observations are collected worldwide, using a variety of platforms and with considerable variability in spatial and temporal resolution. These platforms include:

- aircraft during takeoff and landing, and at flight level,
- satellites providing radiance measurements and cloud drift winds,

- instrumented balloons (radiosondes),
- automated and manual surface weather stations,
- radar,
- ocean and lake-based buoys.

All of these disparate datasets must be organized into an analysis, that is, a representation of the current state of the atmosphere. For the mathematical models, these analyses must be balanced in a manner that is consistent with the type of model, such that non-meteorological signals are not generated in the solutions. Various methods for addressing this have been developed over many decades, but ultimately this consideration combined with inevitable error in the observations means that not all data are actually used in a given analysis. Typically, the resolution of the observations is not as high as that of the model; consequently, a first-guess of the analysis is based upon a prior forecast (typically 6 hours old validating at the start time of the forecast), and is then modified based on adding the observations using these balancing approaches.

Reanalyses use all the available data, retrospectively, including observations that may have not made their way into the operational forecast owing to transmittal delays, and use a fixed model analysis system in a similar manner as above, to produce consistent analyses of the atmospheric state on the models spatial grid for a period of several decades (Kalnay *et al.*, 1996). These reanalyses then become a standard “observational” dataset, which is well sampled (typically on a uniform grid) in space and time, with relatively high resolution in both dimensions, and can be used in weather and climate research studies. In particular, the reanalysis data sets can be utilized in validating forecasts of dynamical models, and typically serve as the input data for purely statistical emulators of atmospheric and climate dynamics (e.g., Kravtsov *et al.*, 2017). These authors point out that “the reanalysis products estimate the path of climate evolution that actually happened, and their use in a probabilistic context — for example, to document trends in extreme events in response to climate change — is, therefore, limited. Free runs of dynamical models without data assimilation can in principle be used for the latter purpose, but such simulations are computationally expensive and are prone to systematic biases.” Kravtsov *et al.* (2017) further demonstrate how these problems can be

alleviated with the use of data driven emulators, with the focus on surface temperature modeling over North America region.

3 Implications for Actuarial Work

In this section, we discuss the rationale behind risk pricing, the interplay between weather extremes and insurance, and several areas of actuarial work that could benefit most from weather and climate modeling. The overall objective of this section is to gain a deeper understanding about, and identify potential areas of collaboration between, two scientific fields – atmospheric sciences and actuarial science – so that the quality of actuarial models and decision making can be improved.

3.1 Risk Pricing

Most of the standard insurance products are sold on a year-by-year basis. The premiums of risks are designed around the expected behavior of the corresponding (random) loss variable and take its mean value as a starting point. This is known as the net or pure premium. Then, to build reserves and prepare for unfavorable deviations from the mean, a safety loading is added to the net premium. The safety loading has to take into account the likelihood and magnitude of “extremes,” as well as other related risks and expenses (e.g., lapse rates, interest rates, underwriting costs). In addition, insurance contracts often have some built-in loss control specifications such as deductibles, policy limits, coinsurance factors, all of which affect the raw random variable in non-trivial ways. There is a large literature on the insurance premium principles, including those based on economic utility theory and distortion risk measures (Young, 2003), that addresses the question on how to compute the net premium and safety loading with a single functional of the underlying probability distribution.

There are also non-standard contracts that can be underwritten for a single event over a specified (usually short) period of time, multi-year contracts, and financial derivatives whose payoff can be triggered by a pre-defined event. The latter might not look like a typical insurance product that is sold to individuals yet they serve as useful risk management instruments for companies and organizations. In any case, the underlying idea for pricing all such products is the same: the expected value of the random variable plus a safety loading (which may be zero as is the case with derivatives). Thus,

in order to compute these risk-pricing components, we need to have a robust estimate of the entire probability distribution for the random variable of interest, which is the frequency and severity of events that trigger a payment over a fixed time interval. Note that the largest payments result not from the part that the net premium covers but from the safety loading part. For example, over time the mean of a random variable may stay constant or even decrease, but if at the same time the frequency of extremes increases that generates more large payments.

3.2 Weather Extremes and Insurance

As discussed in Section 2.2, weather and climate predictions do not directly predict behavior of the random variable which results in an insurance payment. However, weather-related events can have big influence on the parameters of probability model that describes the frequency and severity of payments. Such explanatory variables are known as risk drivers, risk factors, or covariates, and they could be modeled using the weather and climate models. Of course, for insurance purposes the focus should be on the extreme behavior, rather than average, of climate variables. Two types of extreme climate events — *severe but non-catastrophic events* and *natural catastrophes* — require further discussion.

Examples of the first type include droughts, heat waves, cold spells, floods, hailstorms, and snowstorms. The spatial scale of such events is on the order of tens of miles. In other words, these events can be characterized as local. Nonetheless, they do affect agriculture, energy and communications sectors, result in general business interruption and supply chain disruption as well as in significant property damage and even in a loss of life (fortunately, the latter has been decreasing thanks to modern communications and early-warning systems). In the aftermath of such events, the insured losses are covered by individual insurance and reinsurance companies. The probability of local severe weather events can be well characterized by a combination of meteorological dynamical modeling, statistical post-processing and/or purely data-driven empirical modeling, as per the discussion in Section 2.2. Hence, the development of relevant covariates to be used in risk pricing is potentially the most fruitful area of interaction between the actuarial and climate science.

On the other hand, natural catastrophes can be labeled as “extremes of extremes”. In this category we include hurricanes and tropical storms, tornadoes, and wildfires. While the spatial scale of these

events can vary from miles (e.g., tornadoes) to tens of miles (e.g., wildfires) to hundreds of miles (e.g., hurricanes), they all are extremely destructive to property and infrastructure, highly damaging to the environment, and result in a loss of life. Total economic losses from these events measure in billions and even tens of billions of dollars. The direct effect of such events can be local or regional but when it comes to recovery efforts they impact entire industries (not only insurance and reinsurance), capital markets, and state and federal governments. In view of this, Charpentier (2008) questioned whether such events are insurable at all: “natural catastrophes are now hardly insurable: losses can be huge (and the actuarial pure premium might even be infinite), diversification through the central limit theorem is not possible because of geographical correlation (a lot of additional capital is required), there might exist no insurance market since the price asked by insurance companies can be much higher than the price householders are willing to pay (short-term horizon of policyholders), and, due to climate change, there is more uncertainty (and thus additional risk).” Hence, despite recent progress in forecasting the catastrophic events (as well as their response to climate change) using numerical weather prediction models, the utility of these predictions in the actuarial field is unclear.

3.3 Specific Examples

Here we discuss two specific areas where recent developments in climate modeling can add value to actuarial work: *actuaries climate index* and *weather derivatives*.

3.3.1 The Actuaries Climate Index

To better understand the impact of climate change and the importance of climate change to insurers, the leading actuarial organizations of North America – American Academy of Actuaries, Canadian Institute of Actuaries, Casualty Actuarial Society, and Society of Actuaries – sponsored the development of the Actuaries Climate Index (ACI, 2016). There is also a plan to develop the Actuaries Climate Risk Index which by design will measure correlation of economic and human losses by peril to the relevant climate variable.

The ACI is based on actual historical data and intended to be a monitoring and educational tool. In other words, it is a retrospective measure and does not provide projections about future events.

The index tracks changes of *extremes* in six climate-related variables – temperature, precipitation, consecutive dry days, wind, sea level – over time (at monthly resolution), and covers the United States and Canada. All data is standardized to measurements over the reference period of 1961 to 1990. The key metric is a 5-year moving average which was chosen as a compromise between the data noise and climate signal. Currently, a seasonal ACI and monthly indices are published each quarter. They report on the most recent available meteorological season (three months ending February, May, August, and November) compared to the reference period. Users can follow changes in the seasonal ACI and its individual components, for Canada and the United States separately, the Canada-U.S. regions combined, as well as 12 sub-regions.

Note that the ACI is defined to track the past and current changes in extremes relative to the distributions estimated over the 30-year reference period, and thus does not address directly the time-dependent evolution of the actual probability distributions (and extremes) associated with the ACI’s input climatic variables. This probabilistic information can be supplied via ensemble simulations using dynamical or statistical weather/climate prediction models. State-of-the-art high-resolution dynamical models are expensive to run and may suffer from biases (see Section 3.2), and their targeted use for this purpose in the near future is unlikely. In this context, recent work of Kravtsov *et al.* (2017) outlined a proof-of-concept strategy for enhancing the ACI and making it a truly probabilistic index using data-driven, purely statistical emulators of meteorological data. In particular, the authors produced a high-resolution, 100-member ensemble simulation of surface atmospheric temperature over North America for the 1979–2015 period, using the input data from the North American Regional Reanalysis (see Section 2.3) to build their model. This non-stationary statistical model accounts for spatiotemporal interdependencies within the surface-temperature data and incorporates external climatic predictors that describe long-term climate trends. Using various model validation tools, they demonstrated that the temperature realizations generated by this model are statistically indistinguishable from actual historical data. The simulated library of temperatures over North America provides an estimate of complete probability distribution function of surface temperature for any point on the grid (with nominal spatial resolution of 20 miles) and at any point in time (daily minimum and maximum). This simulated data set can replicate all aspects of the temperature variable in the ACI, including

all the local indices, and provides additional information such as estimates of the variability of the frequency of extremes and at a higher resolution. Moreover, while the currently produced simulations are retrospective, just like the ACI, the model can be re-engineered to be predictive. To enhance the whole ACI, additional research would be needed to produce similar kind of simulations for the other five variables.

3.3.2 Weather Derivatives

Many businesses and organizations are exposed to weather-related risks in general and temperature risk in particular. An example of financial losses resulting from temperature extremes can be a utility company experiencing an increase in energy demand due to a warmer than usual summer or colder than usual winter. To meet the demand, the company would have to invest extra money for purchasing and transporting electricity from other locations. One popular approach to managing such a risk is to take a position (i.e., buy or sell) in temperature derivatives, a special class of weather derivatives. In general, as mentioned by Brockett *et al.* (2009), weather derivatives have certain advantages over weather-related insurance products — they offer more flexibility and require no proof of loss.

Payoffs of temperature derivatives are directly linked to temperature indices. Two commonly used temperature indices – cumulative cooling degree days (CDD) and cumulative heating degree days (HDD) – are defined, for a specified location, as a total number of

$$\text{Daily CDD} = \max\{0, T - 65\} \quad \text{and} \quad \text{Daily HDD} = \max\{0, 65 - T\},$$

respectively, where T is the daily average temperature (measured in degrees Fahrenheit). The accumulation of daily HDDs and CDDs is done over a pre-specified contract period, with typical examples being a calendar month, a winter or summer season, or one of the six-month periods April-September or October-March. Six-month contracts are commonly traded, and have been studied by several researchers (e.g., Campbell and Diebold, 2005; Erhardt, 2015).

Although there are various market-based methods for pricing weather derivatives (see Brockett *et al.*, 2009, for a discussion), it seems that actuarial techniques are most appropriate for rare weather events such as extreme heat or cold (Erhardt and Smith, 2014). Using the actuarial approach, the focus would be on estimating the probability of $T > 65$ or $T < 65$. The probability of these events,

however, depends on spatio-temporal interactions of variable T . Several authors have incorporated spatial dependence in their models (Erhardt, 2015, and Hua *et al.*, 2017), but not at a resolution that was achieved in the empirical model of Kravtsov *et al.* (2017) discussed earlier. Once again, the key take-away point here is that ensemble simulations using spatially extended weather and climate prediction models, both dynamical and statistical, can be directly implemented to estimate probability of meteorological events relevant for actuarial modeling and decision making.

4 Summary

In this paper, we have given an overview of the weather and climate system, discussing how the non-linearity and non-stationarity of its multiple, interacting components across a wide range of space and time scales constrains prediction. Next, we have presented the hierarchy of models — from dynamical models based on first principles to purely statistical models solely concerned with accurate representation of the observed meteorological statistics — all of which are used to understand and to forecast this complex system. One important use of these weather/climate prediction models for actuarial studies is to develop dynamically consistent meteorological analyses, which, despite intrinsic limitations owing to their construction, represent the best available, well sampled estimate of the meteorological time series relevant for actuarial applications (e.g., for estimating the Actuarial Climate Index time series). Perhaps most importantly, the available weather/climate prediction models of all types can produce ensemble simulations that can supply probabilistic information to be included in the form of covariates in the actuarial models. This knowledge can be effectively leveraged in constructing the Actuaries Climate Index, in estimating expected payoffs of weather derivatives, and, in similar ways, in addressing any particular risk pricing problem.

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