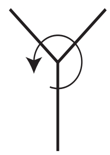
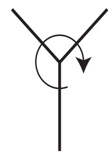
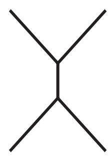
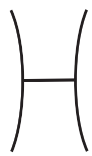
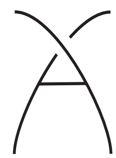
PROBLEM SESSION

1. **James Conant**

$$FD = \frac{\mathbb{Q}\{\text{unitrivalent connected `oriented' graphs}\}}{AS,\ IHX} = \text{`Feynman Diagrams'}$$

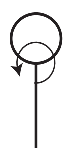
AS:  +  = 0

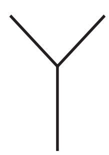
IHX:  =  − 

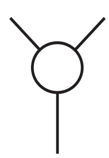
$FD_k = \{\text{diagrams with } k \text{ univalent vertices}\}$

Question: Is $FD_k = 0$ if k is odd?

Example.

 = 0

 = 0

 = 0

2. **Jim Cannon**
(on Heegaard Splittings) Let $J_1, J_2, \cdots, J_k$ be disjoint simple closed curves on a surface S_g of genus g. Let ϕ be the homeomorphism from S_g to itself formed by applying the right hand Dehn twist about each J_i. Sew two copies of the same handlebody of genus g bounded by S_g to one another by the homeomorphism

ϕ. Is it possible to obtain all orientable 3–manifolds by this construction? Is it possible to obtain T^3?

Comment: We believe that it is possible to obtain all orientable 3–manifolds if the twists are allowed to be both right and left handed.

3. **Greg Conner** (in honor of Jack Lamoreaux)
For a Peano continuum X and $x, y \in X$, let

$$M(x,y) = \{ z \mid d(z,x) = d(z,y) \} .$$

We say X has the *Double Midpoint Property (DMPP)* if, for all $x \neq y \in X$, $|M(x,y)| = 2$.

If X has the DMPP, is $X \approx S^1$?

4. **Bob Daverman**
Given a crumpled 3–cube C, can $\mathbb{R}^3$ be tiled with mutually congruent copies of C? (In other words, there is an embedding $\lambda : C \to \mathbb{R}^3$ and all tiles are congruent to $\lambda(C)$.)

Comment: The answer is "yes" is $Bd(C)$ is locally collared at some point.

Same question for crumpled n–cubes tiling $\mathbb{R}^n$.

5. **Bob Daverman**
Given a finite group Γ, does there exist a compact n–manifold M^n ($n = n(\Gamma)$) and a regular covering $\theta : M^n \to M^n$ such that

$$\Gamma \cong \pi_1(M^n) / \theta_{\#}(\pi_1(M^n)) \, ?$$

How about requiring $n = 3$?

6. **Paul Fabel**
Suppose K is a compact [path] connected subset of $\mathbb{R}^2$. Does K have the *Isotopy Extension Property*? That is, given an isotopy $f_t : K \to \mathbb{R}^2$, $0 \leq t \leq 1$ with $f_0 = id_K$, does there exist an 'extension isotopy' $F_t : \mathbb{R}^2 \to \mathbb{R}^2$ such that $F_0 = id_{\mathbb{R}^2}$ and $F_t|_K = f_t$.

Comments: The above is known to be true when X is locally connected. It is false when K is the Cantor set, but true when K is the cone over a Cantor set.

7. **Yusuf Gurtas**
It is known for complex surfaces that $c_1^2(X) \leq 9\chi(X)$. Is there a similar inequality for simply connected, oriented, closed, smooth, symplectic 4–manifolds? In particular, is the above inequality still true?

8. **David Snyder**
   Is there an upper semicontinuous decomposition of a compact, orientable finite dimensional manifold into copies of the solenoid [such that the decomposition space is finite dimensional]? Such that the image is a resolvable generalized manifold?

9. **David Snyder**
   If $\mathcal{G}$ is an upper semicontinuous decomposition of a 4–manifold M into circles, is the decomposition map approximately the orbit map of a locally smooth circle action on M?

10. **Tom Thickstun**
    What open 3–manifolds embed in S^3?

    Comment: The following special case is known:

    Theorem (Thickstun, 1987). *(Assuming the Poincaré Conjecture) An open, connected, orientable,* 1*–acyclic at infinity* 3*–manifold U embeds in S^3 if and only if there exists a degree one map of S^3 onto the end-point compactification of U (which is necessarily a homology* 3*–manifold).*

11. **David Wright**
    Conjecture: Let C be a Cantor set in $\mathbb{E}^3$ that is strongly homogeneously embedded. Then C is tame. (*Strongly homogeneously embedded* means that any homeomorphism $h : C \to C$ can be extended to a homeomorphism from $\mathbb{E}^3$ to itself.)