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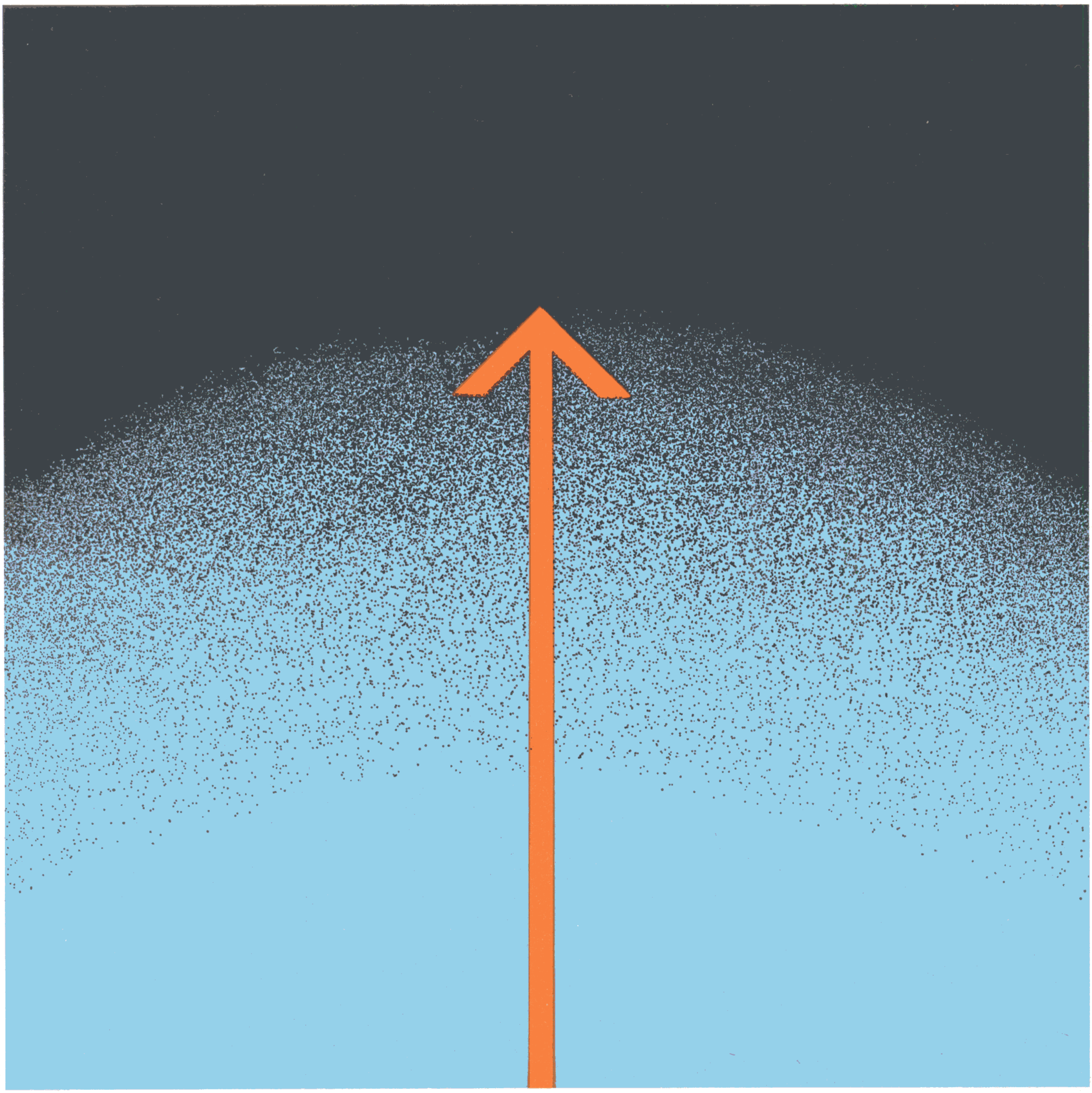

Contributions to Atmospheric Physics

Beiträge zur Physik der Atmosphäre

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Fractal Characterization and Simulation of Lightning

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Abstract:

Analysis of photographs of lightning indicates that lightning has a fractal geometry associated with a reproducible fractal dimension of about 1.34. Following this analysis, a model is presented which generates structures that are qualitatively similar to lightning observed in the atmosphere and that exhibit a fractal dimension of about 1.37. The results from the model indicate that its basic assumptions correctly represent the underlying major physical processes involved.

Zusammenfassung: Der fraktale Charakter des Blitzes und seine Simulation

Die Analyse von Blitzphotos zeigt, daß der Blitz eine fraktale Geometrie mit einer reproduzierbaren fraktalen Dimension von etwa 1,34 besitzt. Im Lichte dieser Analyse wird ein Modell vorgestellt, das Strukturen erzeugt, die qualitativ denen von in der Atmosphäre beobachteten Blitzen ähneln und die eine fraktale Dimension von etwa 1,37 aufweisen. Die Ergebnisse des Modells zeigen auch, daß seine Grundannahmen die beim Blitz wesentlichen physikalischen Prozesse in korrekter Weise enthalten.

Résumé: Caractérisation fractale et simulation d'éclair

L'analyse de photographies d'un éclair indique que celui-ci a une géométrie fractale associée à une dimension fractale reproductible de l'ordre de 1,34. Suite à cette analyse, on présente un modèle qui engendre des structures qualitativement similaires aux éclairs observés dans l'atmosphère et dont la dimension fractale est de l'ordre de 1,37. Les résultats du modèle indiquent que ses hypothèses fondamentales représentent correctement l'essentiel des processus physiques sous-jacents.

1 Introduction

Lightning is the result of dielectric breakdown of gases which occurs when some region of the atmosphere attains a sufficiently large electric charge. Basiscally, a strong concentration of negative charge within the cloud base produces electric fields which cause some negative charge to be propelled towards the ground. This cloud-to-ground discharge is called the stepped leader because it appears to move downward in steps. When the stepped leader has lowered a high negative potential near the ground the resulting high electric field at the ground is sufficient to cause an upward-moving discharge which carries ground potential up the path previously forged by the stepped leader. By doing so the return discharge illuminates and drains the branches formed by the stepped leader. This return discharge is called the return-stroke. Both, therefore, the stepped leader and following return stroke are usually strongly branched downward.

No two lightnings are alike. Lightning comes in an extraordinary variety of structures which appear random (Figure 1). This is the main reason that lightning has defied any quantitative characterization. Yet anybody can distinguish lightning from any other growth form.

Visual examination of lightning photographs reveals a striking presence of structure at many different length scales. Every branch, for example, looks itself like a lightning and so does every branch of a branch. It seemed, therefore, appropriate to attempt to examine the structure of lightning using the

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Figure 1
Cloud-to-ground lightning. Photograph by Arjen *VerKait*. Used with permission. Note that this photograph is a reproduction from a photograph and some of the details are lost.

concept of fractals or self-similarity (MANDELBROT, 1983). A fractal (or a scale invariant structure) is an object whose statistical properties are unchanged under a change of spatial length scale. In other words, two pieces of a fractal, one of size A and the other of a size A' ($A' < A$), are statistically equivalent over some wide range of intermediate lengths, as long as the smaller pieces is magnified by a factor A/A'. In this paper we will present evidence for fractal properties of lightning by analyzing photographs and by the study of a simple theoretical stochastic model of dielectric breakdown.

2 The Fractal Geometry of Lightning

For Euclidean structures the amount of mass, M, scales with some characteristic length, l, as:

$$M(l) \propto l^{d} \tag{1}$$

where d is equal to the spatial or Euclidean dimension of the space in which the structure exists (3 for a sphere, 2 for a plane, 1 for a straight line). For many non-Euclidean objects in nature (clouds, for example) equation (1) is preserved but the exponent is no longer equal to the Euclidean dimension of the space in which the object exists. In these cases $M(l) \propto l^{D}$ where $D < d$ and need not be an integer. These objects are called fractals and D is called the fractal dimension (MANDELBROT, 1983). For self-similar structures the fractal dimension does not depend on l and it is the same for all the structures of a common origin (such as clouds). This is an important result because it implies an order in structures that appear random.

Accordingly, if a lightning is a fractal structure the relation between the total length, L, of all branches inside a circle (or square) of radius (or half side) l, and l itself should be a power law with noninteger D (MANDELBROT, 1983):

$$L(l) \propto l^{D} \tag{2}$$

In order to estimate the fractal dimension of lightning we have applied the above principle to twenty photographs. Most of the photographs were taken from SALANAVE, 1980. The selected photographs exhibit some degree of branching and in our analysis it was assumed that the thickness of the branches is zero and that lightning exists on a plane. The photos were enlarged or reduced to a common size of about 10×10 cm. For each lightning we placed randomly on the photograph a square of a side $2l = 1$ cm (i.e. $l = 0.5$ cm) and we measured the total length (using a steplength of 2 mm) of all branches inside that square. The square was repositioned several times to obtain a mean estimated value for the length, $\langle L \rangle$, for the above value of l. We then repeated the above procedure for a range of increasing

l values up to 5 cm. We then plotted log $<L>$ against log l. This procedure results in a graph which for the above range of l values is almost linear with slope D. The quantity D is the fractal dimension of the structure under examination (MORSE et al., 1985; NIEMEYER et al., 1984).

Our analysis indicated that $D = 1.34 \pm 0.05$. 1.34 is the mean of the twenty estimated fractal dimensions and 0.05 is the standard deviation. One other widely used technique to estimate the fractal dimension of an object is the following: Take a big square of side set to 1 which includes the object. Then pave it with subeddies of side $r = 1/2$, and find the number of squares, N, which intersect the object. Repeat the above process with subsubeddies of side r^2. Continue as far as feasible. The number N scales as a function of r according to the relation $N \propto r^{-D}$ where D is an estimation of the fractal dimension of the object (MANDELBROT, 1983). When we applied the above technique to the photos we obtained the same result: $D = 1.34 \pm 0.05$. Therefore, lightning is a fractal with a reproducible fractal dimension of about 4/3. Such a result provides for the first time a quantitative characterization of lightning. Having such a characterization, we can now present a nonequilibrium model which simulates fractal structures which are quantitatively and qualitatively similar to lightning. A model is called a nonequilibrium one when randomizing effects dominate stabilizing (deterministic) effects.

3 A Model for the Simulation of Lightning

The model employed here for simulating lightning is a modification of the nonequilibrium model proposed by NIEMEYER et al. (1984) for the modeling of two-dimensional radial discharge commonly referred as Lichtenberg Figures. The details of the model used here are as follows: the simulation is a stepwise procedure carried out on a two-dimensional lattice (Figure 2) in which the potential (ϕ) of the top and bottom row is fixed at a value of $\phi = 0$ and $\phi = 1$ respectively. Periodic boundary conditions are assumed at the sides of the lattice. Only the middle point of the top row (A_1) is capable of growth. Given these initial conditions the potential at every point of the lattice is obtained by solving the Laplace equation $\nabla^2 \phi = 0.0$. On a two-dimensional lattice this is obtained by iterating the following equation using successive over relaxation (SOR):

$$\phi_{i,j} = \frac{1}{4}(\phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j+1} + \phi_{i,j-1}) \tag{3}$$

All the immediate non-zero neighbors to point A_1 are then considered as possible candidates, one of which will be added to the evolving discharge pattern. In Figure 2 the candidates are indicated by open circles and the evolving pattern by black dots. In Step 1 there is only one possible candidate. Therefore point A_2 will be added to the discharge pattern, which is considered equipotential ($\phi = 0$). In step 2 one solves again the Laplace equation taking into account that the boundary conditions should include

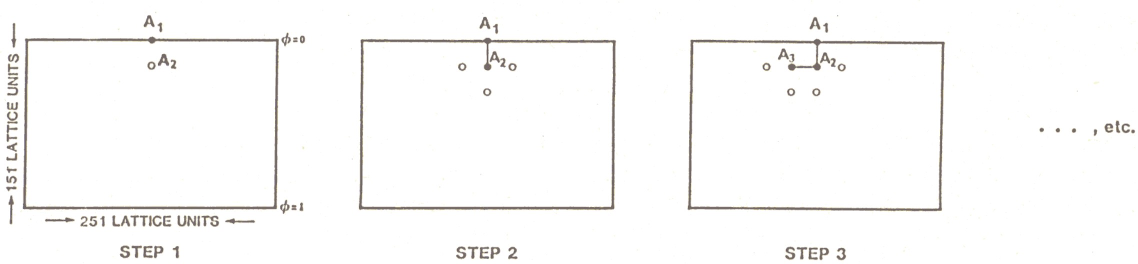


Figure 2 Illustration of the model used to simulate lightning. The discharge pattern is indicated by the black dots, connected with solid lines and it is considered equipotential.

The open circles indicate the possible growth sites. The probability of each one of these sites is proportional to the local potential field (see text for details).

the discharge pattern. The possible candidates in Step 2 are three and each one of them is assumed to be associated with a probability P (commonly called "growth" probability) which is defined as:

$$P_i = \phi_i^2 \Big/ \sum_{i=1}^{N} \phi_i^2 \qquad (4)$$

where i = 1, ..., N and N is equal to the number of the possible candidates. Apparently in Step 2 N = 3. **The growth probability depends, therefore, on the local field determined by the equipotential discharge pattern.** At each step a probability distribution is defined. Given this probability distribution a point is randomly selected and added to the evolving pattern. The above procedure is then repeated until the first point of the bottom row is added to the discharge pattern. Figures 3, 4 and 5 show three examples of computer generated lightning. In order to obtain good convergence when iterating (3) we performed 50 interactions at each step. Early experimentation indicated that after 50 iterations the potential field on each possible growth site changes by around 0.1 % per iteration which indicates very good convergence. The problem, however, it that the program becomes very laborious and expensive. Each lightning simulation (~ 700 points) takes on a UNIVAC 1100 computer about 3.5 hours in CPU time!
The above model reproduces very effectively the influence of a given discharge pattern on the growth probability of each candidate. For example, the tip of the line (indicated by A in Figure 3) will have a larger growth probability than points inside a cage (indicated by B in Figure 3). These are the well known "tip effect" and "Faraday screening" which result from the solution of the field equation (NIEMEYER et al., 1984) and it is obvious from the results that the model produces structures which unquestionably look like lightnings.

Apart from the fact that the model produces structures qualitatively similar to lightning it is necessary to verify whether or not these structures are also quantitatively similar to lightning. In order to do this we have calculated the fractal dimension of the five computer generated lightnings. A widely used method to determine the fractal dimension of computer simulated nonequilibrium growth structures is the following. For each point of a given structure we determined the number, $n(l)$, of all the other points within a square of half side l (l is now measured in lattice units) and its average $< n(l) >$ over all the points of that structure, for varying l. We then plotted $\log < n(l) >$ again $\log l$. The structure is a fractal if for a relatively wide range of scales the graph is approximately linear with a slope D. The quantity D is called the correlation dimension (WITTEN and SANDER, 1981; HENTSCHEL and PROCACCIA, 1983; LOVEJOY et al., 1986) and again it provides an estimation of the fractal dimension of that structure.

Figure 3

Figure 4

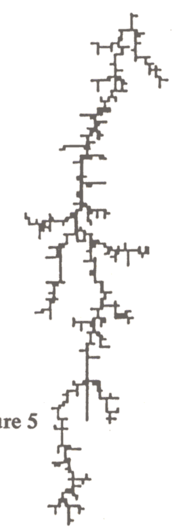
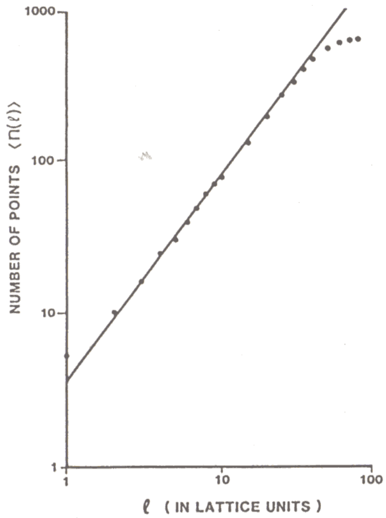


Figure 5

- **Figure 3**
  An example of a computer generated lightning. This structure is made up from 726 points.

- **Figure 4**
  An example of a computer generated lightning. This structure is made up from 592 points.

- **Figure 5**
  An example of a computer generated lightning. This structure is made up from 666 points.



• **Figure 6** For the structure in Figure 3, this graph shows the average number of points $< n(l) >$ within a square of increasing half side l. The $l^{1.36}$ function is shown for comparison. It can be observed that for the range of l values between 2 and 50 the variation of $< n(l) >$ with l is almost linear with a slope D = 1.36. Therefore, the estimated fractal dimension of the structure in Figure 3 is 1.36.

Figure 6 shows $\log < n(l) >$ versus $\log l$ for the structure shown in Figure 3. It can be observed that for a considerably wide range of l values (from 2 to about 50), $\log < n(l) >$ varies linearly with $\log l$. The $l^{1.36}$ function is also shown for comparison. One may, therefore estimate the fractal dimension of the structure in Figure 3 as about 1.36. From the five simulations we obtained that $D = 1.37 \pm 0.02$. This is very close to the value of 1.34 ± 0.06 derived from the photographs. The larger standard deviation is probably due to the finite size and resolution of the photos which do not allow a higher accuracy in determining D. In addition to the above described method there are other techniques which are commonly used in order to estimate the fractal dimensionality of computer simulated non-equilibrium growth structures (MEAKIN, 1986). For example, the mass (number of occupied lattice sites) M(r) contained within the distance r measured from the initial growth site, scales with r according to the relation $M(r) \propto r^{D'}$ where D' is an estimation of the fractal dimension of the structure. According to this method we obtained that $D' = 1.35 \pm 0.02$. In the asymptotic limit (where the range of length scales becomes infinite) D and D' should converge to a common value.

The above results indicate that in addition to producing structures that are qualitatively similar to lightning the employed model produces structures that are also quantitatively similar to lightning. It should be mentioned at this point that the above model simulates the stepped leader process. The photographs on the other hand most likely represent return strokes because only these are luminous enough. The stepped leader takes some milliseconds to reach the ground whereas the return stroke propagates upward in a fraction of a millisecond. However, since the return stroke propagates upward on the path previously forged by the stepped leader we expect the geometry of the stepped leader to be identical to the geometry of the return stroke.

4 Some Additional Comments and Results from the Model

Since the model provides the probability of each candidate at each step one may calculate the probability of the whole structure by multiplying the probabilities that all the selected candidates were associated with. For the structure in Figure 3 this probability is $10^{-1,075}$ (!) This indicates that to see the same lightning is virtually impossible and explains the great variety in the patterns of lightning. In a perfectly uniform atmosphere one would expect the breakdown to spread out in a straight line. As a matter of fact according to the model a straight line lightning is the most probable one. However, because there is always some noise in the system a growth instability will occur and irregularities will appear (NITTMANN and STANLEY, 1986). In our case this noise can be density or temperature or humidity fluctuations, for example. This noise is reproduced in the proposed model via the random selection procedure at each step. There is always a chance that a site with a very small probability will be selected. Thus, the evolving structure soon becomes very irregular.

One may wonder about the choice of the exponent in (4). This exponent may vary from 0.0 to d, where d is the Euclidean dimensionality of the space in which the growth process is embedded. It has been demonstrated by NIEMEYER et al. (1984) that the fractal dimension of the generated structures depends on that exponent. For larger values of the exponent the generated fractal structures tend to be more "linear" and the fractal dimension is smaller. What justifies the choice of the exponent in this study is the fact that for a value of the exponent equal to two we simulate structures that are qualitatively (look like) and quantitatively (same D) similar to observed lightning.

One final note. In our simulations we have used a rectangular lattice. This is only a reflection of our feeling that lightning takes place in a three dimensional space where the z direction is smaller compared to the x or y direction. Prelimanary experimentation indicates that the consideration of a square lattice or a different rectangular lattice will not significantly affect the results.

5 Conclusions

We have examined the fractal properties of lightning from photographs and have presented results from a dielectric breakdown model that generates structures which are qualitatively and quantitatively similar to lightning. The major conclusions from this study are two: 1) lightning is fractal with a reproducible fractal dimension of about 1.37 and 2) lightning can be simulated assuming a growth probability which depends on the local potential field which is determined by the equipotential discharge pattern. Such an assumption naturally leads to fractal structures which are consistent with photographs of observed lightning.

We already know that cloud-to-ground discharge pattern is a stepwise procedure. The assumptions of the model suggest that at each step the evolving pattern modifies the potential field thus affecting the possibilities of its evolution constantly. The success of the employed model may indicate that its basic assumptions capture the essence and the principles behind the underlying processes that produce lightning in the atmosphere.

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